A MARXIST-JAMESIAN APPROACH TO THE AXIOM OF CHOICE

JEANNE CHERVONNA

ABSTRACT. I will argue against the set theoretic Axiom of Choice on the basis of a progressive form of the Jamesian will-to-believe principle.

Note: I wrote this pseudonymous spoof, inspired by the Sokal affair, when I was a first year graduate student at Pittsburgh. For the humor impaired (or if I failed at humor!): *this is a spoof*. I haven't changed it except to fix some typos and the like. – Alexander Pruss

1. INTRODUCTION

The Axiom of Choice (AC) states that given a collection $\{U_i\}_{i\in I}$ of disjoint non-empty sets U_i, it is possible to *choose* an element out of each of the sets U_i, i.e., there is a function f on I such that $f(i) \in U_i$ for all $i \in I$. It is well known that the Axiom of Choice is logically independent of the axioms of Zermelo-Fraenkel (ZF) set theory; within ZF, it is neither possible to prove AC nor to disprove AC. Ought we then to believe AC? William James has given us a suggestion on how one should proceed in cases in which one can neither prove nor disprove something. In such cases, we have a *will-to-believe* situation—the person should believe that option which has the best consequences for her.

James' principle is, of course, tainted to some degree by the individualistic capitalistic ethic of his upbringing, but it has a point of validity. The natural way to modify his principle to fit with the ideals of social progress is to say that in will-to-believe situations, i.e., situations in which neither a proposition nor its negation can be proved, one should choose to believe that option belief in which is optimal in terms of progressive societal consequences. I will call this the *James-Pravda*

Date: April 1, 1997.

principle, as extreme versions of this principle have been widely applied in the respected progressive newspaper *Pravda*.[1]

The James-Pravda principle applies directly to AC given that AC can neither be proved nor disproved. Thus, to determine whether AC should be accepted, we need only to consider societal consequences. I will argue that in fact AC is a highly reactionary principle. It will immediately follow that AC ought to be rejected. This, I believe, is the first time a philosophically sound argument has been presented as to why AC should or should not be believed. The failure of philosophers of mathematics in the past to stumble on this argument has doubtless been due to their petty bourgeois roots, and due to their neglect of the James-Pravda principle.

2. Why the Axiom of Choice is reactionary

2.1. **Abortion.** It has recently been suggested that Axion of Choice is in fact a *progressive* element, because it can be linked with women's rights to choose abortion.[2] This argument is heavily flawed as it neglects the essential mathematical feature of the Axiom of Choice.[3] The Axiom of Choice is a non-constructive claim. It does not provide any *effective procedure* for effecting the choice, i.e., for finding the choice function f on I; we cannot in general write down a specific choice function. Thus, the Axiom of Choice should instead be linked to the position of those closet conservatives who say that choice is *possible* and who keep abortion nominally legal, but who do not provide those in need with an effective procedure for exercise of the choice. The Axiom of Choice is thus a sop to the feminist movement; it does not provide real choice, but only an ineffective nominal semblance of a choice. Providing such semblances of choice, similarly to those provided in such ultra-conservative initiatives as *school choice*, is a special bourgeois strategy for attacking progress by replacing it with ineffective theoretical progress.

[1]It is worth noting, however, that *Pravda* had applied a stronger principle to the effect that *in general* one should believe the proposition which is better in terms of the progress of society, and not just in cases where neither the proposition nor its negation can be proved. However, this more general principle is more controversial, and for my argument I only need the weaker principle.

[2]See the final endnote of Alan Sokal, "Transgressing the boundaries: Towards a transformative hermeneutic of quantum gravity", *Social Text*.

[3]And it is worth noting that Sokal's work has been generally discredited, and he himself has withdrawn his support for his arguments, so that we need not worry about Professor Sokal's authority in this regard.

2.2. **Multiplication of capital.** Furthermore, the Axiom of Choice betrays its rootedness in the industrial revolution in other ways. For instance, it is a central capitalist myth that through trade, capital can be rearranged in such a way that it might grow. This myth finds strong support in the Axiom of Choice. For suppose we have a single golden sphere. Then, the Axiom of Choice via the Banach-Tarski theorem implies that one can decompose the sphere into a finite number of disjoint pieces, and moving the pieces around with rigid motions, one can reassemble them into two spheres, of the the same radius as the original one. In other words, through financial transactions, one has multiplied the amount of capital by two. Given that the ordinary working class person knows very well that it is not possible to multiply a single sphere of gold into two spheres of gold by trade and rearrangement, the capitalist must invent an ingenious argument to show that this can be done, and thus to confuse the revolution class instinct of the working classes. The capitalist's ingenious argument, as we have seen, depends crucially on the Axiom of Choice. Doubtless no sane member of the working class will fall for it, but some intellectuals have, unfortunately, fallen for it, and thereby social progress has been impaired. But for us who are aware of the James-Pravda principle, this argument only shows that the Axiom of Choice ought to be rejected. It is important to note that once the Axiom of Choice is rejected, the Banach-Tarski theorem can no longer be proved—capital no longer can be multiplied by capitalist sleight-of-hand.

2.3. **Centrality of the economy.** It is a crucial premiss in Marxist thought that everything in human activity is centered on economy, and thus that we can measure all human activity in numerical economic terms. Reactionary logicians, however, have tried to argue against this measurability of all human activity. They have shown that there exist sets on the *real line*[4] which are non-measurable in the technical mathematical sense.[5] This provides powerful support for the neo-conservative striving to distract the masses from economic issues by pointing out that there are allegedly things which cannot be measured in economic,

[4]It is a particular strength of the reactionary argument that the sets are purely on the *real* line, and do not have any *imaginary* component such as religion. It is evident that if one allows for such imaginary components—for such "opium of the masses", as Marx said—then examples of non-measurable, i.e., non-quantifiable, sets can be found. Of course, a non-measurable set would have to be non-existent, since something that exists cannot lack extension, and something with extension cannot lack measure, but of course the reactionary "logicians" neglect this.

[5]Walter Rudin, *Real & Complex Analysis*, Third Edition, New York: McGraw Hill, 1987, p. 53.

i.e., quantitative, terms. However, the reactionary logicians in their arguments have been assuming the Axiom of Choice. Once this unjustifiable Axiom is rejected, it no longer follows that there is anything non-measurable. All things can be quantified, and non-economic issues cannot distract us from the economic exploitation of the working classes. Therefore, we have another argument via the James-Pravda principle for rejecting the Axiom of Choice.

2.4. **Elitism, competition and "winning".** Reactionary societies seek to maximize the lot of elites, while leaving the working classes to support them. In effect, for a reactionary society, it is the elites that count, and the structure of society is defined by them. The average person is neglected by capitalists, who care only about the extremely rich and who, also, sometimes make a show of care for the *extremely* poor, while neglecting the great masses of working people who are exploited in between. In mathematical terms, this is closely related to the Krein-Milman theorem which says that the it is the extreme points of a compact convex subset of a locally convex space which can be used to define the whole of that compact convex subset (indeed, this compact convex subset is just the closed convex hull of the extreme points). In fact, even the weaker claim that there *exist* extreme points has adverse social implications, for it shows that it is impossible to have a societal system (under assumptions of compactness and convexity, of course) in which all are equal, without extremes. The neglect of the importance of the *average person* whose presence can simply, for the capitalist, be inferred by interpolation between extremes, is a grave obstacle to social progress, and we have thus seen that the Krein-Milman theorem provides significant ammunition for the reactionary position. However, if we dig more deeply, we find that the Krein-Milman theorem depends very heavily on the Axiom of Choice. Indeed, the Krein-Milman theorem (and even the weaker claim that there exist extreme points in the setting of the Krein-Milman theorem) is logically *equivalent* to the Axiom of Choice.[6] Hence, the adverse social consequences of adopting the Krein-Milman theorem provide us with serious considerations against the Axiom of Choice.

Moreover, the Axiom of Choice is equivalent to Zorn's Lemma, which says that certain maximal elements in certain sets must exist. Again, the existence of maximality—indeed, also the whole obsession with maximality, competition and "winning" between elements—is central to reactionary capitalist dogma.

[6]J. L. Bell and D. H. Fremlin, "A geometric form of the axiom of choice", *Fundamenta Mathematicae* **77**, 167–170, 1972.

2.5. **Religion and Platonism, versus constructivism.** Capitalist society has introduced *religion* and *idealism*, which distract the working classes from *material* concerns. The Axiom of Choice is an important party in this deception. The Axiom, let us recall, tells us that there is a certain function f on a set I, a function *which cannot be constructed*, but which can only be posited. Evidently, such a function cannot be said to exist in any *material* sense, since in a material sense all there exist are *effective procedures*. Therefore, the existence of such a function is contrary to the basic principles of Marxist materialism.

Acceptance of the existence of such a function leads to *idealism*, and indeed eventually to mathematical *Platonism*. We know, however, how reactionary Platonism is, in its positing of supersensible forms and in its claims of the unimportance of changing matter. It is, furthermore, deeply misogynic, in its hatred for the *feminine* quality of matter. Finally, contemplation of the "eternal forms" leads one to forget about the cares of this changing world, and thus is a pastime merely of the rich.

Platonism and the Axiom of Choice are directly contrary to societal progress. The Axiom of Choice rules out constructivism, and thus it leads to think that there are structures which are not simply social constructs—for, indeed, the "choice" function given by the Axiom of Choice is not a social construct (indeed, the function is non-constructible). However, it is a central premiss in social progress that *all* social structures are social *constructs* and thus can be changed at will to best promote class equality. Hence, once again, we see the reactionary nature of the Axiom of Choice.

But one of the most pernicious issues involved in the Axiom of Choice is its connection to *religion*. It is well known that religion is the ultimate enemy of progress. It is the "opium of the masses". It distracts the working person from her duty of class warfare, and instead tries to substitute mythical heavenly ends. The strongest opposition, to social progress in matters such as abortion comes precisely from religious persons, usually in fact from religious *men*.[7] How then does the Axiom of Choice relate to religion? Well, first of all, let us recall Zorn's Lemma, which proves that the Axiom of Choice is logically equivalent to the claim that in certain partially ordered sets there exist maximal elements. Such non-constructible, hence immaterial, maximal elements

[7]That Sokal (*ibid.*) has missed the connection between the Axiom of Choice and religion, and hence the radical way in which the Axiom of Choice is *opposed* to choice in abortion, shows in another way that Sokal's work was the result of an interpellation by capitalists seeking to protect their interests from the deep critique of capitalist science in which the respected journal *Social Text* has engaged.

are precisely the same kind of thing as the supernatural gods of various religions. Moreover, there have been philosophers who have used the Axiom of Choice in more specific support of religious dogma. For instance, Robert Meyer[8] has argued that the Axiom of Choice is logically equivalent to the existence of God.[9] This should immediately alert us to the reactionary and socially harmful import of the Axiom. Even more recently, I have heard that a graduate student in my own Department has used, admittedly among other arguments, the Axiom of Choice to defuse a powerful objection of Hume and Edwards to the Cosmological Argument for the existence of God.[10]

3. Conclusions

The Axiom of Choice thus has deeply socially reactionary implications in many areas. Since it is known that the Axiom of Choice can neither be proved nor disproved, we are in a Jamesian will-to-believe situation. A Marxist-Jamesian thus must apply the James-Pravda principle to conclude that the Axiom of Choice ought not be held. Much military-funded mathematics has been done on the basis of the Axiom of Choice. Many more mathematicians have become inadvertently attached to the Axiom because of a lack of attention to its social implications. The present paper is the first solid progressive argument for rejecting the Axiom of Choice.

By the later work of James in which his beneficial-to-believe criterion is extended from simply being an *epistemic* norm to being a definition of *truth*—an extension which is most reasonable given that everything is socially constructed as we anti-AC (but, of course, pro-genuine-choice) constructivists know—it follows that in fact the Axiom of Choice is *false*, and hence a long standing mathematico-logical controvery is now settled.

Department of Philosophy, University of Pittsburgh, Pittsburgh, PA 15260, U.S.A.

[8] "God exists!", *Noûs*, **21**, 345–361, 1987.

[9] Meyer uses a Cosmological-type Argument to prove that if the Axiom of Choice holds, then God exists. Conversely, Meyer says that if God exists, then God can choose the choice function by means of divine omnipotence.

[10] Alexander R. Pruss, "The Hume-Edwards Principle and the Cosmological Argument", preprint.