A GÖDELIAN ONTOLOGICAL ARGUMENT IMPROVED
EVEN MORE

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1. Introduction

Gödel’s ontological argument axiomatizes the notion of a positive property, and then argues, based on plausible further “non-formal” axioms about which properties are in fact positive, that there is a being that has at least some of the central attributes of God.

The formal axioms are:

F1. If $A$ is positive, then $\sim A$ is not positive.

F2. If $A$ is positive and $A$ entails $B$, then $B$ is positive.

There are several ways of metaphysically understanding the notion of a positive property.\(^1\) On the excellence view, a positive property is one that in no way detracts from its possessor’s excellence, but whose negation does. On the limitation view, a positive property entails no limitation in its possessor, but its negation does. According to Leibniz, there are basic properties, all subsets of which are mutually compatible. Leibniz would probably define a positive property as one that is a conjunction of basic properties.\(^2\) But we can modify his view to be open to the possibility that some basic properties are not valuable, by saying that some basic properties are excellences and a positive property is one that is entailed by one or more basic properties that are excellences. Each of these accounts makes F1 and F2 very plausible.

Now consider the following two non-formal axioms:

N1. Necessary existence is positive.

N2. Essential omniscience, essential omnipotence and essential perfect goodness are positive properties.

We can then define $A$ to be a strongly positive property provided that $EA$, the property of having $A$ essentially, is a positive property. (By F2, strongly

\(^1\) Pruss (2009) also offers an account due to Maydole on which a property is positive provided that it is better to have that property than not to have it. However, Oppy (2009) has rightly pointed out that it is far from clear whether the disjunction of a property that it is better to have than not to have with a property that it is better to not have than to have counts as something that it is better to have than not to have.

\(^2\) Along these lines, Pruss (2009) makes the suggestion that one could take a positive property to be one that is entailed by one or more basic properties.
positive properties are positive.) Pruss (2009) then assumes a modal logic including S5 and proves:

**Theorem T1.** Given F1, F2 and N1, if \( A \) is a strongly positive property, then there is a necessarily existing being that essentially has \( A \).

It follows from F1, F2, N1 and N2 that there is a necessarily existing being that is essentially omniscient. And one that is essentially omnipotent. And one that is essentially perfectly good. But Pruss (2009) could not show, without making further controversial assumptions, that there is a being that has all these three essential properties.

This paper remedies this defect. Admittedly, we will make further non-formal assumptions, but they will be very plausible.

We will end up by discussing a reformulation of the arguments in terms of negative properties as well as Oppy’s parody of the Pruss (2009) argument.

2. **Uniqualization**

There can be at most one being that has the property of being the tallest woman. If \( A \) is a property such that it is impossible that there exist \( x \) and \( y \) such that \( x \) and \( y \) each have \( A \), but \( x \neq y \), then we shall say that \( A \) is uniqualizing. Being the tallest woman is uniqualizing.

The following non-formal axiom is very plausible:

**N3.** There is at least one uniqualizing strongly positive property.

In fact, it is very plausible that we can give an example of one. Axiom N3 follows from N2 and:

**N4.** Essential omnipotence is uniqualizing.

Why should we think essential omnipotence is uniqualizing? Well, the idea of two essentially omnipotent, or even two contingently omnipotent, beings is deeply problematic. Omnipotence requires perfect freedom and an efficacious will.\(^3\) But there cannot be two beings with perfect freedom and an efficacious will. For if they are perfectly free, they will be able to will incompatible propositions to be true, and then one of their wills shall have to fail to be efficacious. (This argument assumes that we are individuating beings in such a way that distinct beings with will have their own will. If God is a Trinity, the persons of the Trinity do not have distinct wills, and hence will not count as distinct beings in our sense.)

Other plausible examples of an equalizing strong positive property are being greater than every other being as well as being creator of every other being. To work with the latter property, we can assume:

**N5.** Being essentially such that one is creator of every other being is a positive property.

**N6.** If \( x \) is creator of \( y \), then \( y \) is not creator of \( x \).

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\(^3\)Pearce and Pruss (forthcoming) argue that omnipotence just is perfect freedom and efficacious will.
And then being creator of every other being will be strongly positive and uniqualizing, so that N3 will follow.

All in all, it does not appear that N3 is very controversial.

We can now add to the results from Pruss (2009):

**Theorem T4.** Given F1, F2, N1 and N3, it follows that there is a necessary being that essentially has every strongly positive property.

The following immediately follows:

**Corollary C3.** Given F1, F2, N1, N2 and N3, there is a necessary being that is essentially omnipotent, essentially omniscient and essentially perfectly good.

Now, define:

**Definition D4.** A God is a being that essentially omnipotent, essentially omniscient, essentially perfectly good, and creator of every other being.

Then:

**Corollary C4.** Given F1, F2, N1, N2, N5 and N6, there necessarily exists a unique God.

For N5 and N6 implies N3, so that by T4 it follows that there is a God, and by N6 this God is unique, as he is creator of every other being.\(^4\)

The proof of T4 needs the following lemma from Pruss (2009):

**Lemma L1.** Given F1 and F2, any pair of positive properties is compossible.

(The proof is easy: if \(A\) and \(B\) are positive and not compossible, then \(A\) entails \(\sim B\), so that by F2, \(\sim B\) is positive, and by F1, \(B\) cannot be positive as well, so absurdity follows.)

To prove T4, let \(U\) be a uniqualizing strongly positive property, by N3. By T1, there is a necessarily existing being that essentially has \(U\). Let’s say that Umberto is such a being. Let \(A\) be any strongly positive property. Then \(EA\), the property of having \(A\) essentially, is also positive. By L1, \(EA\) is compossible with \(U\). Thus there is a possible world \(w\) at which there is a being, \(x\), that has both \(EA\) and \(U\). But Umberto exists at every world and has \(U\) at every world. Thus, Umberto exists at \(w\) and has \(U\) there. Since \(U\) is uniqualizing, it follows that \(x\) is identical with Umberto. Therefore, Umberto has \(EA\) at \(w\). Thus, it is true at \(w\) that Umberto necessarily has \(A\). By S5, it follows that at the actual world it is also the case that Umberto necessarily has \(A\). Hence, Umberto essentially has \(A\). Thus, Umberto essentially has every strongly positive property.

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\(^4\)Some think (mistakenly, I believe) that God is not the creator of abstract entities, and indeed that it is impossible for any being to be the creator of abstract entities, but that there nonetheless necessarily exist abstract entities. Impossible properties aren’t going to be positive, since an impossible property entails non-positive properties like *being cruel*, so if it’s not possible to be the creator of abstract entities, and there must be abstract entities, we will instead need to work with the property of being the creator of every other *concrete* being. Plausibly this property is strongly positive, and if we assume the axiom that every creator is concrete, we conclude that this property is strongly uniqualizing.
It is interesting to note that the above argument also shows that for every positive property \(A\), even ones that are not strongly positive, Umberto at least possibly has \(A\). For by L1, \(A\) is compatible with \(U\), and hence at some world some being has both \(A\) and \(U\). But only Umberto has \(U\) in that or any other world, and so Umberto has both \(A\) and \(U\) in that world.

3. Negative properties

There is, perhaps, something somewhat unnatural and gerrymandered about the notions of positive properties offered in the introduction. The notion of a positive property on all three accounts offered does not exactly correspond to any intuitive notion of an excellence or a property that’s worth having. For instance, since anything entailed by a positive property is positive, if \textit{knowing that} \(2+2=4\) is a positive property, so is \textit{being foolish or knowing that} \(2+2=4\).

This disjunction is not a counterexample to F2 given the three stipulative metaphysical accounts of positive properties offered in the introduction, but given the relative complexity of the stipulations, our intuitions about the non-formal axioms are liable to be less confident than we would like.

We might, however, proceed differently, by taking as our primitive the notion of a negative property, which is actually more natural than the G"odelian notion of a positive property. We can think of a negative property as one that \textit{limits} a being in some way. The following two axioms then are intuitively plausible:

\textbf{F1*}. If \(A\) is negative, then \(\sim A\) is not negative.

\textbf{F2*}. If \(B\) is negative and \(A\) entails \(B\), then \(A\) is negative.

Axiom F1* tells us that to lack something that limits one is not limiting, i.e., that limitation is avoidable, while F2* tells us that a property that entails a limitation is limiting.

We can then stipulate a positive property as one whose negation is negative. It is easy to see that under this stipulation the conjunction of F1 with F2 is equivalent to the conjunction of F1* and F2*. But it is better to work with the more natural notion of a negative or limiting property as in F1* and F2*.

We can define a \textit{strongly negative} property as a property \(A\) such that \(PA\) is negative, where \(PA\) is the property of possibly having \(A\). A property is strongly negative if and only if its negation is strongly positive. We can say that a property is \textit{nearly universal} provided that it must be had by all except at most one being. A property is nearly universal if and only if its negation is uniqualizing.

We still need some non-formal axioms. These will be direct translations of the N-axioms. For instance:

\textbf{N1*}. Possible non-existence is negative.

\textsuperscript{5}Cf. Oppy’s criticism of Maydole’s definition of positivity (Oppy, 2009, p. 359).
Possible non-omniscience, possible non-omnipotence and possible lack of perfect goodness are negative properties.

There is at least one strongly negative nearly universal property.

These axioms are still plausible. To substantiate N3∗ we just use the negations of our examples of candidates for strongly positive uniquequalizing properties (or small variants on them, if desirable for stylistic or intuitive reasons). Thus coexisting with an entity one did not create or not being omnipotent are properties that, plausibly, all beings except at most one can have.

Moreover, given F1∗ and F2∗, the starred versions of the N-axioms are equivalent to the unstarred versions. Hence we get equivalent starred versions of our results T4, C3 and C4. For instance, we have:

**Corollary C4.** Given F1∗, F2∗, N1∗, N2∗, N5∗ and N6∗, there necessarily exists a unique God.

The only difference between C4∗ and C4 is that the argument behind C4∗ did not rely on a gerrymandered concept of positivity.

Let us go back to the initial counterintuitiveness of the idea that being foolish or knowing that $2 + 2 = 4$ is positive. The parallel claim on the side of negativeness is that not being foolish and not knowing that $2 + 2 = 4$ is negative, i.e., limiting. And that certainly is true—it is limiting through its second conjunct.

### 4. Oppy’s Parody

Oppy begins by stipulating the notion of a *natural* property (not in the sense in which in the preceding section I talked of some notions as more natural than others) as

a property whose instantiation in no way entails the existence of any supernatural entities, or the holding of any supernatural states of affairs, or the like, but the instantiation of whose negation does in some way entail the existence of supernatural entities, or the holding of supernatural states of affairs, or the like. (Oppy, 2009, pp. 360–1)

Oppy then offers two formal axioms about naturalness. The first parallels F1, and the second parallels a plausible generalization of F2. Then Oppy offers a non-formal axiom:

The following property is natural: having no world-mate that is a necessarily existent, essentially omnipotent, essentially omniscient, essentially perfectly good being (Oppy, 2009, p. 361).

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6Oppy’s text has a reduplicated “essentially omnipotent” in several places. I assume one of the occurrences is always meant to be “essentially omniscient.”
Oppy argues that his axioms entail that there is no necessarily existent, essentially omnipotent, essentially omniscient and essentially perfectly good being. And he’s right about that.

One problem with Oppy’s parody is that there his notion of a natural property is quite unnaturally gerrymandered, arguably even more so than the three versions of the notion of a positive property considered in the introduction, and much more so than the one in the preceding section which was defined in terms of negativity. One way in which Oppy’s notion of a natural property is gerrymandered is that it is defined in an extrinsic way—a natural property is one whose negation entails that there are supernatural states of affairs, rather than entailing something about the entity that has it. This gerrymandering makes the intuitions behind his non-formal axiom less reliable.

A second problem is that given Oppy’s stipulative definition of a natural property, the mere assumption that there are any natural properties is incompatible with classical theism. For according to classical theism, God is a necessarily existent and essentially supernatural being. But if $A$ is a natural property in Oppy’s sense, then the instantiation of $A$ does not entail (I am reading Oppy’s “in no way entail” as just “does not entail”) the existence of any supernatural entities, and hence the proposition that $A$ is instantiated does not entail the existence of God. But if God exists, then every proposition entails his existence. So the very existence of a natural property is incompatible with theism.

This gives the classical theist a simple reason to reject Oppy’s non-formal axiom that his complex property about lacking divine world-mates is natural, because the classical theist is committed to there not being any natural properties in Oppy’s sense.

But there does not appear to be a parallel point to be made about the notions of positive and negative properties, since it is highly intuitive, whether or not God exists, that there are some positive properties, such as knowing that $2 + 2 = 4$ or such as not being cruel, and some negative properties, such as not knowing that $2 + 2 = 4$ or such as being cruel. The very idea that there is a positive or a negative property does not by itself appear to commit one to anything that an atheist rejects. Thus Oppy’s parody is dialectically inferior to these Gödelian arguments.

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7One might worry whether God counts as supernatural in worlds in which there is no nature. But one might reasonably respond that in those worlds, God is trivially supernatural, being trivially beyond the realm of the natural, because there are no realm of the natural there.

8What if the natural property were non-instantiable? But my argument did not assume instantiability. Moreover, there cannot be any non-instantiable natural properties, because if $A$ is a non-instantiable property, then the proposition that $A$ is instantiated entails every proposition, and in particular entails the proposition that there is a supernatural being, contrary to Oppy’s definition of naturalness.
5. Conclusions

The improved Gödelian arguments of Pruss (2009) can be improved some more by introducing the notion of a uniqualizing property. Moreover, instead of running the arguments with the somewhat gerrymandered notion of a positive property, one can instead take as primitive the more natural notion of a negative or limiting property. Finally, Oppy’s (2009) parody is not parallel to these theistic arguments, because Oppy’s notion of a “natural property” is such that the mere assumption that there is a natural property is incompatible with classical theism, which makes Oppy’s parody dialectically inferior to the argument that it is a parody of.

References