

DEFENDING DAMNATION

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ABSTRACT. Some people have the intuition that the doctrine of damnation—that some people reach a point where the possibility of their being saved and receiving eternal union with God is foreclosed—is incompatible with the existence of a perfectly good and generous God. I shall argue to the contrary by offering toy models that are plausibly compatible with the existence of such a God but where nonetheless a significant number of people can be expected to be damned. I then argue that the models can be tweaked while maintaining their compatibility with theism in such a way that they become models of the doctrine of hell, which adds to the doctrine of damnation that the foreclosure happens at death, that the life of the damned continues forever, and that there is on balance pain in that life.

1. INTRODUCTION

Let us say that the doctrine of damnation holds that some people become *damned*, i.e., at some point in their existence, the possibility of their being saved and receiving eternal union with God is conclusively foreclosed. The doctrine of damnation is a consequence of the traditional Christian doctrine of hell which adds three additional ingredients: the point in time at which the possibility of salvation sometimes gets foreclosed is death, the damned person continues existing forever, and the damned person on balance suffers

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pain during that existence. But one can also have doctrines of damnation that fall short of the doctrine of hell. Perhaps there is some additional time after death during which a person might repent. Perhaps the damned are annihilated. Perhaps they do continue forever, but in a painless neutral state.

In this paper, I will offer an argument that the doctrine of damnation is compatible with the existence of an omnipotent, omniscient, and perfectly good God who is very generous to all people. I will also offer some reason to think that the existence of such a God is also compatible with the doctrine of hell as defined above, but my main focus will be on the doctrine of damnation.

There are three main alternatives to the doctrine of damnation among theists. First, we have necessitarian universalism, on which God eventually necessitates everyone to be saved. Second, we have contingent libertarian universalism, on which the contingent free and indeterministic choices of people are such that everyone is saved, but they could have been such as to make some of them be damned. Third, we have infinite chances views, exemplified perhaps by Lewis's *Great Divorce* GD and more recently by Gwiazda [9], on which God always gives more chances for choosing salvation to people who have turned down the previous ones.

Of the three alternatives, the contingent libertarian universalist theist presumably agrees with my conclusion that the doctrine of damnation is compatible with the existence of a very generous theistic God—it's just that contingently the doctrine is false. Thus the main *relevant* alternatives

to the doctrine of damnation will be necessitarian universalism and infinite chances.

Rooney [15, p. 13] has recently argued against universalist implications of infinite chances models by noting that there can be a non-zero probability of rejecting God infinitely many times provided that after rejection one becomes sufficiently less likely to accept God. However, he has not analyzed whether the degree of character deterioration an infinite chances model would need to posit in order to yield a non-zero probability of eternally rejecting God is reasonable to posit. Indeed, Gwiazda [9, footnote 7] has already suggested that such a model would be implausible.

I will argue that a certain reasonable infinite chances model of salvation—the Infinite Chances Toy Model—is compatible with the existence of a very generous theistic God. I will then argue that there is a model of damnation that is no less plausibly compatible with the existence of such a God. Finally, I will give some reason to think that the compatibility claim remains plausible even if we further specify the model of damnation in such a way that it becomes a model of the doctrine of hell.

2. TOY MODELS

2.1. The Toy Model Schema. I describe a Toy Model Schema about what happens at death. The Toy Model Schema will turn into several models, depending on how various parameters are filled out.

We have a number of people—beings relevantly similar to ourselves—and they all eventually die. Some people at that point have a character state sufficiently good that they have already been redeemed and will receive the

eternal goods of happiness, virtue and union with God. Others, call them “Wicked”, will instead receive repeated gracious offers of eternal salvation of a certain specific type—call each such offer a “Grace”. The Wicked person has an indeterministic free choice, independent of their past choices or the choices of other people, whether to accept or reject the Grace. If they accept it, their character is repaired by God, and they receive the eternal goods of happiness, virtue and union with God. When a Grace is rejected, their character deteriorates somewhat. The probability of accepting a Grace is always non-zero, but is lower the worse the character is. Unless and until a Wicked person receives the Grace, they live in a state without union with God or positive bliss, in a hedonically neutral state.

As part of our Toy Schema, I take it that we can measure the quality of the character of a person by the probability that they would accept a Grace. Furthermore, I take it that there is a deterioration coefficient δ strictly between 0 and 1 such that if a person with character quantified by a Grace acceptance probability p rejects the grace, they lose a fraction δ of the quality of their character. In other words, their Grace acceptance probability goes from p to $(1 - \delta)p$.

The number of offers of Grace that God is willing to give each Wicked person after death will be denoted by N_G , which may be infinite or finite. The total number of people is fixed at N_P . There is an upper cut-off U_W for Wickedness, where a person is Wicked provided the character they die with has a probability of Grace acceptance less than U_W . There is an independent probability w that any given person will be Wicked, and hence we expect wN_P people to be Wicked. Among the Wicked, character quality

at death (as measured by Grace acceptance probability) is uniformly distributed between a lower bound $L_W \geq 0$ and our upper Wickedness cut-off U_W , both exclusive. No one can manage to be as Wicked as L_W .

We thus specify a particular model in the Toy Model Schema by assigning numbers to the following:

- (i) N_P : number of people (positive integer)
- (ii) U_W : upper Wickedness bound (strictly between 0 and 1)
- (iii) L_W : lower Wickedness bound (non-negative and less than U_W)
- (iv) w : probability of Wickedness (strictly between 0 and 1)
- (v) δ : deterioration coefficient (strictly between 0 and 1)
- (vi) N_G : number of offers of Grace (positive integer or infinity)

Now, suppose a Wicked person dies with a character whose quality is measured by p_1 , the probability of Grace acceptance. They have a probability $1 - p_1$ of rejecting Grace. If they do that, their character will deteriorate to one whose quality is quantified as $p_2 = (1 - \delta)p_1$. They will now have a probability $1 - p_2$ of rejecting Grace. If they do that, their character will in turn deteriorate to

$$p_3 = (1 - \delta)p_2 = (1 - \delta)^2 p_1.$$

And so on. If they reach an n th offer of Grace, their probability of rejection will be $p_n = (1 - \delta)^{n-1} p_1$. The probability r that they will reject *all* offers of Grace then depends on their character quality at death p_1 , the deterioration coefficient δ , and the number of offers N_G , and is equal to the product of

N_G of the numbers p_n . If N_G is finite, this is:

$$r(p_1, \delta, N_G) = (1 - p_1)(1 - p_2) \cdots (1 - p_{N_G}) = \prod_{n=1}^{N_G} (1 - (1 - \delta)^{n-1} p_1)$$

and in the infinite case this is:

$$r(p_1, \delta, \infty) = (1 - p_1)(1 - p_2) \cdots = \prod_{n=1}^{\infty} (1 - (1 - \delta)^{n-1} p_1).$$

This is a known mathematical function, called the q -Pochhammer symbol (where q corresponds to our $1 - \delta$) or the q -shifted factorial, and denoted $(p_1; 1 - \delta)_{N_G}$, with N_G finite or infinite. Obviously, the probability $r(p_1, \delta, N_G)$ of always rejecting decreases as the number of offers N_G increases. However, it is easy to prove that if $p_1 < 1$, then $r(p_1, \delta, \infty) > 0$, and hence even given infinitely many offers, there is a non-zero probability that a Wicked person will reject them all.¹

We can also calculate the probability that a randomly chosen Wicked person will always reject grace, i.e., will be always unsaved. This is

$$(1) \quad R = R(\delta, N_G, L_W, U_W) = \frac{1}{U_W - L_W} \int_{L_W}^{U_W} r(x, \delta, N_G) dx.$$

For the character quality p_1 at death of the person is uniformly distributed from L_W to U_W . Finally, we can calculate the expected number of people in the model who persist in rejection of God:

$$N_R = w N_P R.$$

¹Taking logarithms, we have $r(p_1, \delta, \infty) > 0$ if and only if $\sum_{n=1}^{\infty} \log(1 - (1 - \delta)^{n-1} p_1) > -\infty$. For sufficiently small positive x , we have $\log(1 - x) \geq -2x$. Thus, for n sufficiently large $\log(1 - (1 - \delta)^{n-1} p_1) \geq -2(1 - \delta)^{n-1} p_1$. But for $\delta > 0$, we have $\sum (1 - \delta)^{n-1} < \infty$, and hence we have $\sum_{n=1}^{\infty} \log(1 - (1 - \delta)^{n-1} p_1) > -\infty$.

We can use the formulas above for calculations using Mathematica software, which provides `QPochhammer` $[a, q]$ (infinite case) and `QPochhammer` $[a, q, n]$ (finite and infinite n) functions, as well as the `NIntegrate` numerical integration formula. For instance, to calculate the probability $r(p_1, \delta, N_G)$ of the rejection of N_G offers for someone who starts with a probability p_1 of acceptance of Grace, with a deterioration coefficient of 0.01, and with N_G chances, we evaluate

$$\text{QPochhammer}[p_1, 1 - \delta, N_G].$$

On the other hand, to calculate the probability $R(p_1, \delta, N_G, L_W, U_W)$ of a random Wicked person always rejecting grace, we evaluate:

$$(1/(U_W - L_W)) * \text{NIntegrate}[\text{QPochhammer}[x, 1 - \delta, N_G], \{x, L_W, U_W\}].$$

2.2. Common parameter choices. One specifies a toy model by filling in the numbers in the abstract Toy Model Schema. Some sets of numbers may not appear to be compatible with God's goodness, generosity, and respect for free formation of character, but others do appear to be. We first fill in some details about numbers that all our Models will have in common.

If the deterioration coefficient δ is very close to zero, the person's choice to reject Grace does not affect their character much at all. This has an obvious advantage, namely that their character does not deteriorate much. But it also has a serious disadvantage, namely that the person is unable to control their character very effectively. On the other hand, if δ is close to one, the person has significant control over their character in each choice, but the price is heavy: they can very easily spiral down and down. Such an ability for quick spirals may seem incompatible with God's goodness.

I suggest that it is compatible with God's goodness and generosity that δ be around 0.01. It may seem that a 1% loss of character per rejection of God's grace is too small to allow significant control over character, but a daily bad choice over a thirty days will deteriorate one's character by $1 - 0.99^{30}$, or approximately 26%, quite a significant change. So I take it that a value of δ near 0.01 is both compatible with God's goodness and generosity, and gives us significant control over our character.

Thus, in all the models I will consider, we will have δ near (but not always exactly equal to) 0.01.

In all the models, I will take the upper Wickedness cut-off to be $U_W = 1/2$. Thus, if a person dies with a character that is more likely than not to accept Grace, God takes them into eternal bliss (presumably repairing their character, perhaps through a purgatory-like process). If they die with a character that is more likely than not to reject Grace, the person is Wicked, and gets N_G posthumous offers of grace. And because God is generous, if anybody's character happens to be right on the line at $1/2$, they get eternal bliss.

I will fairly arbitrarily stipulate there are 10 billion people, so $N_P = 10^{10}$.

In all the models, I will also take it that over the course of life, God offers opportunities for character growth such that most people do not end up Wicked at the end of life. In fact, in all the models, I will stipulate that the probability of Wickedness is only $w = 0.001$. Thus, we can expect $wN_P = 10^7$, or ten million, people to be Wicked.

Finally, I assume that God makes sure that people do not deteriorate too far in this life, so that nobody ends life with a character whose grace acceptance probability is 0.05 or lower. In other words, $L_W = 0.05$, and so the character of the Wicked at death, as measured by grace acceptance probability, is uniformly distributed between 0.05 and 0.50. Preventing Wicked characters at 0.05 or worse, while nonetheless allowing free character formation to result in a full range of Wicked characters between 0.05 and 0.50 seems to strike a reasonable balance of benevolence and respect for freedom.

Only two numerical parameters have not been fully specified yet, and the models we will consider will vary with respect to them. First, we have the deterioration coefficient δ , which I said would be near 0.01, but I did not give a precise value. Second, we have the number of offers N_G .

3. THE INFINITE CHANCES TOY MODEL

On the Infinite Chances Toy Model, the number of offers is $N_G = \infty$. Furthermore, I specify that the deterioration coefficient is *exactly* 0.01. One cannot fault God for his generosity in the number of offers, and the deterioration coefficient is in a range I argued was reasonable in light of God's goodness and respect for freedom. I will also specify that there is a non-zero minimum temporal spacing between offers—say, a week, a day, or an hour. This assumption ensures that at every future time it is true that more offers will be available (the assumption will be dropped in Section 6.1).

We can now use equation (1) and Mathematica to calculate the probability that a randomly chosen Wicked person will never accept any offer:

0.0001362.² Figure 1 summarizes the Infinite Chances Toy Model, to four significant figures in the last two lines.

Number of people	N_P	10^{10}
Upper Wickedness bound	U_W	0.50
Lower Wickedness bound	L_W	0.05
Probability of Wickedness	w	0.001
Deterioration coefficient	δ	0.01
Number of offers	N_G	∞
Expected number of Wicked	wN_P	10^7
Probability of Wicked person never being saved	R	0.0001362
Expected number of unsaved Wicked	$wN_P R$	1362

FIGURE 1. The Infinite Chances Toy Model

Notice that despite infinitely many offers being made, of the 10 million Wicked people, we can expect over a thousand to persist in their Wickedness for eternity and never be saved. Such persistence is a feature of the model's respect for human freedom.

4. RESPONDING TO GWIAZDA

Gwiazda [9] has noted that if the probability of choosing to stay in hell is constant and less than one, then the probability of rejecting infinitely many opportunities is zero. In footnote 7, he has additionally observed that if this probability “very rapidly approaches 1”, then the probability of remaining in hell can be non-zero. However, Gwiazda continues:

²Mathematica input: `1/(.5-.05)*NIntegrate[QPochhammer[x,1-.01,Infinity],{x,0.05,0.5}]`. The Mathematica inputs for further calculations will be omitted.

These are remarkably strong conditions, and I have serious doubts whether they would ever be met by any person P, though the situation is not logically impossible. But the situation is not likely enough to alter the fact that I would bet that any P would leave hell, and I would place this bet at any odds.

Yet a 1% deterioration of character per rejection of Grace does not seem a “remarkably strong” condition. Indeed, it seems quite a reasonable condition. Character is indeed formed by choice through habituation. And this 1% condition suffices to ensure that there is a far from negligible probability that one of the Wicked will continue to remain unsaved despite an infinite number of opportunities. As proved in footnote 1 above, as long as the deterioration coefficient δ is bigger than zero and the starting probability of Grace acceptance p_1 is less than one, we have a non-zero probability of rejecting infinitely many opportunities. Granted, if $\delta > 0$, then the decay in character on our models will always be exponential. But it can still be a very slowly decaying exponential. I think few would say that supposing the quality of one’s character to decrease by one part in a trillion—i.e., by the fraction $1/10^{12}$ —per rejection of Grace would be a “remarkably strong” condition on the decay, and yet this decay is sufficient to guarantee that there is a non-zero probability of staying away from God forever (albeit an incredibly tiny one if p_1 is not too small³). To bet against such rejection “at any odds” on orthodox decision theory one would need to think that there

³Mathematica does not have enough precision to calculate $r(0.25, 10^{-12}, \infty)$, but it does give $r(0.25, 10^{-3}, \infty) \approx 5.7 \times 10^{-117}$.

is literally zero probability that God has set up an infinite chances system similar to a model with $N_G = \infty$, $\delta \geq 10^{-12}$ and $p_1 < 1$, and that seems quite implausible.

5. TWO OBJECTIONS

Some readers may think that infinite chances models with non-zero probability of eternal rejection of God are unacceptable because a perfect God could not respect our freedom to such a degree as to allow created persons any non-zero probability of staying away from union with him forever.

There are two versions of this objection. On one version, we would expect God to simply override our freedom once we have rejected enough offers of Grace, and force us into salvation. I do not have much to say to a reader with this intuition, beyond some standard remarks. It does not appear to fit well with love to force another into love and union with oneself—or at least it does not fit as well as allowing them to freely enter into that love and union, with the real possibility of rejecting that entrance. And it doesn't count as a real possibility of rejecting that entrance if it is inevitable that one would be made to enter if one rejects it.

Another version is more moderate and perhaps more challenging. Instead of God overriding our freedom, God could make it be that there is zero probability of rejecting infinitely many offers, even though there is a non-zero probability of rejecting any specific offer, and even though negative character formation is still possible. For imagine that God has so set up the constitution of persons that when they reject his first offer, their character quality (as measured by probability of accepting Grace) goes from p_1 to $p_1/2$.

Upon second rejection, it goes from $p_1/2$ to $p_1/3$. Upon third rejection, it goes from $p_1/3$ to $p_1/4$. And so on. The decrease is no longer the result of multiplying by a constant $1 - \delta$. The probability of endless rejection is

$$(1 - p_1)(1 - p_2) \cdots = \prod_{n=1}^{\infty} \left(1 - \frac{p_1}{n}\right),$$

which turns out to be zero if $p_1 > 0$.⁴

Notice, however, that on this modified model the agent comes to have less and less ability to form their character by choosing to reject the Grace. The ratio between the state of character after $n + 1$ rejections and the state of character after n rejections approaches unity: $p_{n+1}/p_n = (1/(n + 1))/(1/n) = 1 - (1/(n + 1))$. It is not at all implausible that God's character is compatible with allowing one a greater degree of self-formation of character than that. But more seriously, even though on this model there is a metaphysical possibility of always rejecting God, because that possibility has zero probability⁵ it does not appear to be a genuinely significant amount of freedom. It is still probabilistically inevitable that in the end one turn to God. And if the Wicked agent knew the setup then their knowledge that with probabilistic inevitability they would turn to God would make

⁴An infinite product converges to zero provided that the sum of the logarithms of the terms converges to $-\infty$. But $\log(1 - p_1/n) \leq -p_1/n$, and $\sum_{n=1}^{\infty} -p_1/n = -\infty$.

⁵A fair coin can land heads every time when tossed an infinite number of times, but the probability of its doing so is zero in classical probability—see also the philosophical arguments of Williamson [17]. Some have disputed that this probability is zero, opting for a non-zero infinitesimal Weintraub. If they are right, then likewise in the modified model there is non-zero infinitesimal probability of endless rejection. I doubt they are right, but I suspect that what I will say about the zero probability case generalizes to the infinitesimal case anyway.

them see their freedom to rebel as very hollow indeed—and it would still be hollow if they didn't know.

Note, too, that in responding to this objection it is not necessary to argue that God *has to* respect our freedom to such a degree as to make for a non-zero⁶ probability of endless rejection. All that I need for the argument is that God *could do so* compatibly with his goodness and generosity. We could see God balancing desiderata in his decision, including both his benevolent desire that created persons have beatitude with him forever and his respect for the freedom of love, and all that I need to argue is that the balancing specified in the Infinite Chances Toy Model is possible given God's perfection.

A second objection is to ask why an omniscient and perfectly good God would create an individual that he foreknows will reject him infinitely many times. An easy way out of this is to embrace open theism, denying God's foreknowledge of future contingents. But open theism has typically been rejected by the major monotheistic religions, and there is a better move.

For suppose we say that God relied on his knowledge that Alice will reject him infinitely many times in deciding not to create Alice. Then we have contradicted ourselves. For if God does not create Alice, then Alice won't reject him even once—Alice won't do anything if she doesn't exist. And if Alice won't reject God even once, then God doesn't know that Alice will reject him infinitely many times, since no one knows something that isn't true. Thus, the objection complains that God didn't do something that is simply logically impossible.

⁶And non-infinitesimal.

But the objection can be improved. Instead of asking why God would create an individual he knows *will* reject him infinitely many times, one can ask why God would create an individual he knows *would* reject him infinitely many times if created. But to assume that there is a fact such as that Alice would reject God infinitely many times, given that these rejections are indeterministic (we have, after all, assumed them to have probabilities strictly between 0 and 1), requires Molinism. Moreover, it requires a specifically problematic part of Molinism. For it requires God to be able to rely on his knowledge of what someone would do if created to decide whether to create them. This, as Adams [1] has noted, appears to induce a circularity in the order of explanation. For suppose that God decides to create Bob in part because Bob *wouldn't* reject him infinitely many times. Then Bob's choice for God is explanatorily prior to the truth of the subjunctive conditional *s* that if he were created he would choose for God; but God's decision to create Bob is explanatorily prior to Bob's choice for God, and God's knowledge of *s* is explanatorily prior to that decision, while *s* itself is explanatorily prior to God's knowledge of it. This results in a vicious circle in the order of explanation.⁷

In any case, it is worth conceding that if Molinism and decisions based on Molinist conditionals of free will can be defended, then our argument is in trouble. The argument is also in trouble given theistic compatibilism. If God can allow us to freely choose salvation while yet determining that we will do so, there is good reason to think that he would determine everyone to freely choose him. But this is a very controversial view of freedom.

⁷See also [6] for a critique and [5] for an alternative argument along similar lines.

6. FROM ALWAYS REJECTING GRACE TO DAMNATION

6.1. **Using a supertask.** I have argued that the Infinite Chances Toy Model is compatible with God's goodness, generosity, and respect for freedom. However, that model is one where there is no damnation. At every time at which there is someone unsaved, infinitely many chances are still going to be available, even if one can expect over a thousand people to reject all of them.

But there is a simple way to turn the Toy Model into one where there is damnation. Just make the infinitely many chances occur in a supertask. The first chance occurs half a year after death. The second chance occurs a quarter of a year later, while an eighth of a year later there is a third chance, and so on. The n th chance is $1 - (1/2)^n$ years after death. But a year after death all the infinitely many chances have run out. Of course, to let the agent deliberate whether to accept the Grace, God may have to miraculously accelerate the functioning of the mind to compensate for the shorter spacing between offers.

This supertask story doesn't change the probabilities. We still expect over a thousand people to refuse all the chances. And after that they are damned. And the reasons for thinking that the Infinite Chances Toy Model is compatible with God's goodness and generosity seem to largely carry over to the supertasked version of the model, and hence we have a defense of the damnation doctrine.

Consider now two objections. First, it seems that living forever after the supertask has finished has no point to it, given that there is no longer a chance at the union with God that theists often think is the telos of our

lives. However, note that the supertask model as specified so far does not that there is eternal life for the damned after the supertask of chances. Those impressed by the pointlessness objection can simply suppose that people live for a finite amount of time (say, a week, a year, or a decade) in order to live out the character they have chosen for themselves, and then are annihilated. The damnation doctrine is compatible with such annihilationism.

Without annihilationism, supposing instead continued existence, at least three other responses are possible. The first is to emphasize the value of living by a character one has chosen, a value that can be extended over eternity. The second is to invoke the Augustinian thesis—a consequence of the privation theory of evil—that existence is always good. The third is to leave to the creativity of the reader to think of meaningful activities that could be available to the individuals despite them being so horribly morally marred.

The second objection is that a number of people think supertasks are impossible, because they violate not just the finitism defended by Craig [?], but also the weaker doctrine of causal finitism defended by Koons [11] and Pruss [14], since the life after all the choices is causally affected by all of them. This limits the appeal of the argument. However, there is still significant dialectical value to this argument. First, there have historically been many prominent theists who denied causal finitism: Thomas Aquinas, for instance, famously argued that God could have created a world with an infinite sequence of humans [3, I.46.2]. Second, an agnostic or atheist can be expected to be skeptical of causal finitism, given the support that causal finitism gives to the Kalām argument. If the supertask defense of damnation

works for an agnostic or atheist, it will show them that they do not have as strong a reason as they may have thought to reject theistic religions that accept the damnation doctrine.

6.2. Without a supertask. Suppose now one is impressed by the no-supertasks objection to the supertasked version of the Infinite Chances Toy Model. We can instead consider a different modification to the original non-supertasked Infinite Chances Toy Model, without relying on anything metaphysically problematic. Simply suppose that God simply cuts off the number of posthumous chances at salvation at some finite count, for concreteness letting it be $N_G = 3000$ (about eight years of daily offers), and freezes the person's character at the quality it has at that point, which constitutes damning them. Call this the First Damnation Toy Model. This has one obvious disadvantage, one less obvious potential disadvantage, and two advantages in comparison with the Infinite Chances Toy Model. The obvious disadvantage is that the chance of being saved for those who die in their wickedness is slightly lowered.

The advantages are these. In Infinite Chances, we would expect some of those who rejected grace 3000 times to continue rejecting it for eternity—indeed, we expect over a thousand people to be like that. In continuing such rejection, they would continually incur two harms. First, their character would continue to deteriorate by 1% per rejection. As Socrates taught, deterioration of character is a very serious harm. Second, they would become guilty of one serious sin per rejection—with there being infinitely many such rejections! And we learn from our conscience that being guilty of a serious sin is a very great harm indeed. Thus, freezing their character and ceasing

to make further offers has two advantages, namely the prevention of further deterioration and that of further sins.

However, there is also another potential disadvantage in this procedure. Following Plantinga [13], call the freedom in a choice “significant” provided that one available option is wrong and another is not. The potential disadvantage then is this. Significantly free choices that have an impact on character may have a value in themselves, even if the choices are wrong and the impact is negative, and God’s cutting off the offers after 3000 takes away this value.

Are the disadvantages sufficient to outweigh the advantages, and make the First Damnation Toy Model unacceptable in light of God’s goodness and generosity? I will argue in the negative.

Begin by considering the loss of significantly free choices. Most or all of the value of a morally significant free choice lies in its enabling the possibility of making a significantly free *right* choice. When the choice made is *wrong*, the value in the morally significant free choice as such is either small or non-existent, and is in any case easily outweighed by the disvalue of the choice being wrong. We can see this outweighing from the ways in which it is reasonable for a person to remove unnecessary temptations from their own path and that of others in their care. A parent who knows that a child is in a mood in which they are extremely likely to disobey a command will typically prudently avoid issuing the command, unless there is very good reason otherwise. The mere fact that issuing the command will bestow the value of making a significantly free choice on the child, when it is extremely likely that the choice made will be wrong, is clearly outweighed by the

disvalue of the child's becoming guilty of a wrong choice. This is *a fortiori* true when the wrong choice can be expected to have a negative impact on character, but the judgment remains reasonable even when no further deterioration of character is expected.

Now, we can expect there to be over a thousand people in our models who would go on to make an infinite number of wrongful grace rejection choices if God doesn't stop making the offers after a finite amount, and some others might make a finite number of such wrongful choices beyond 3000. Thus, considering the two-fold disadvantage in both making bad choices and having one's character deteriorate, which is at most slightly compensated for by the intrinsic value of significantly free choices as such, we see that on our First Damnation Toy Model, we have an infinite benefit to about a thousand people over the Infinite Chances Toy Model.

However, we still have the crucial disadvantage: the lowered chance of salvation due to cutting off the offers of salvation after 3000. How much is this chance of salvation lowered by? To that end, we can re-do the Mathematica calculations from the Infinite Chances Toy Model. The result is shown in Figure 2.

A perhaps surprising thing is that except for the N_G line and the caption, Figure 2 is exactly the same as Figure 1. The probability of a Wicked person never being saved is 0.0001362 and the expected number of unsaved Wicked is 1362 in both. What happened? Surely the cut-off has to make a difference.

And indeed the cut-off does make a difference. But because all the input numbers in the models are so arbitrary, it would have been ridiculous when I was preparing the tables to give more significant figures than I have done in

Number of people	N_P	10^{10}
Upper Wickedness bound	U_W	0.50
Lower Wickedness bound	L_W	0.05
Probability of Wickedness	w	0.001
Deterioration coefficient	δ	0.01
Number of offers	N_G	3000
Expected number of Wicked	wN_P	10^7
Probability of Wicked person never being saved	R	0.0001362
Expected number of unsaved Wicked	$wN_P R$	1362

FIGURE 2. The First Damnation Toy Model

the R and $wN_P R$ lines, and so following standard procedure, I rounded the outputs, in this case to four significant values. What if instead of rounding I use the full precision that Mathematica has for the calculations (namely nearly 16 significant figures)? Then in the Infinite Chances Toy Model, the probability of a Wicked person never being saved would have been:

$$R(0.01, \infty, 0.05, 0.50) \approx 0.0001361564641431523$$

and the expected number of unsaved Wicked persons would have been 1361.564641431523, while on the First Damnation Toy Model, we would have

$$R(0.01, 3000, 0.05, 0.50) \approx 0.0001361564641432177,$$

with the expected number of unsaved Wicked being 1361.564641432177. The difference in R is approximately $6 \cdot 10^{-17}$ while the difference in the number of expected unsaved Wicked is around $6 \cdot 10^{-10}$.

There is a difference here, but it does not appear to be at all philosophically significant. Nor should we be surprised that the difference is small if we notice that after 3000 rejections of God's grace, if the initial character has a probability p_1 of grace acceptance, after 3000 rejections, that probability has shrunk to $(1 - 0.01)^{3000}p_1 \approx (8 \cdot 10^{-14})p_1$, which is at most around $4 \cdot 10^{-14}$ since for the Wicked in our models, $p_1 < U_W = 0.5$. And the probability of rejecting infinitely many offers with a starting character quality of $4 \cdot 10^{-14}$ and infinitely more chances is $r(4 \cdot 10^{-14}, 0.01, \infty) \approx 0.999999999996$.⁸

But one might object. What is at stake is the joy of an infinitely long future life with God. A difference of $6 \cdot 10^{-17}$ in the probability of such a life is an expected utility difference of $6 \cdot 10^{-17} \cdot \infty = \infty$.

I have two responses to this. The simple response is to hope in the reader's intuitions. Consider careful readers who considered my argument that the Infinite Chances Toy Model is compatible with God's goodness and generosity. Such readers presumably looked at Figure 1, and may have also noticed the observation I made in the text that the last two lines in the figure are given to four significant figures. I doubt any reader at this point thought: "I am not sure that this is compatible with God's goodness and generosity. To be sure, I need more significant figures." Would it not be absurd to suppose that had the reader learned that the R value with more significant figures was 0.0001361564641431523 they would have agreed with me about the compatibility claim, while if they had instead learned that it was 0.0001361564641432177, they would have balked?

⁸Mathematica warns about potential loss of precision in this calculation.

The difference indeed does not appear significant, and it appears that what we have is a significant overall advantage in the First Damnation Toy Model stemming from about a thousand people being stopped from an infinite number of sins, and an insignificant overall disadvantage from the slight decrease in salvation probability for the Wicked.

But if the reader is not convinced, I have another argument. Consider the Second Damnation Toy Model. This differs from the First Damnation Toy Model in exactly one respect: the deterioration coefficient is 0.00999 instead of 0.01000. Let's run the calculations, with results in Figure 3.

Number of people	N_P	10^{10}
Upper Wickedness bound	U_W	0.50
Lower Wickedness bound	L_W	0.05
Probability of Wickedness	w	0.001
Deterioration coefficient	δ	0.00999
Number of offers	N_G	3000
Expected number of Wicked	wN_P	10^7
Probability of Wicked person never being saved	R	0.0001353
Expected number of unsaved Wicked	$wN_P R$	1353

FIGURE 3. The Second Damnation Toy Model

Notice that now the differences from our Infinite Chances Toy Model do show up, in the third significant figure of the probability of a Wicked person never being saved and the expected number of the unsaved. However, these differences consist in an *increase* in the probability of salvation. The slight decrease in the deterioration coefficient from 0.01000 to 0.00999 has (by

far) outweighed the effect of capping offers of grace at 3000, resulting in an expectation of saving about eight⁹ more people.

Consider someone who thought that the Infinite Chances Toy Model was compatible with God’s goodness, generosity and respect for freedom, but now denies that the Second Damnation Toy Model was also compatible with these. If so, the objection cannot have to do with the probability of salvation: that is *improved* in this capped model. But perhaps one could object that decreasing the deterioration coefficient lessens our ability to form our character. Indeed, it perhaps does, but surely not to a significant degree. In Section 2.1, I was careful to say that a deterioration coefficient *around* 1% would give us significant control over our character. And surely no reader said to themselves: “Well, yes, but only if it’s very, very close to 1%. I might accept 0.999999%, but 0.999% is unacceptably low.” And, in any case, setting $\delta = 0.00999999$ would still make the resulting 3000-chances model have a slightly higher probability of salvation for the Wicked than the original Infinite Chances Toy Model did!¹⁰

Finally, consider one more objection. The argument was based on the idea that rejecting the offer of Grace was sinful. But perhaps it is morally permissible to reject such offers and hence not sinful.

There are, however, at least three reasons why we could suppose it to be sinful in our Toy Models without endangering their compatibility with God’s character. First, it is ungrateful to reject an offer of something good without

⁹Due to round-off error, the more exact figure of eight does not quite match the 1362 – 1353 you would get from the tables.

¹⁰The rejection probability for the Wicked is $R(0.00999999, \infty, 0.05, 0.50) \approx 0.0001361556392186782$.

good reason. Sometimes, of course, we do have good reason to reject offers of good things. If we are offered something good for us but our acceptance of it would impose a cost on the giver or a third party, that gives us reason to reject the offer. We can suppose such a cost to be absent from our Models. Second, as I specified in Section 2.1, the persons in question are beings like us. As Augustine famously noted, our hearts are restless until they rest in God Ch. 1[Augustine] This is a central feature of our existence, and it appears to be very much consonant with God's goodness that the persons he makes have this feature. To refuse the Grace is to refuse to rest in God, to prevent a being with dignity from living out its telos. This is wrong, even if that being is ourselves. If we think—though I believe we shouldn't—that we cannot wrong ourselves, we should at least think that in doing this we wrong the God who has made beings like us with that as their goal. Third, we might suppose that in order to better motivate the persons to accept the Grace, God offers it in the form of a command, thereby creating additional reasons for acceptance, and we do owe obedience to God.¹¹

¹¹Evans [8] has argued that gratitude gives us decisive moral reason to obey God. Murphy [12] has argued that there is no natural duty of obedience to God. This paper is not the place to argue against Murphy in detail. But it is worth noting that Murphy proceeds by arguing that standard ways of reducing the moral reasons for obedience to other moral reasons—like those of gratitude—fail. Even if this succeeds, it does not question the possibility that the duty to obey God could be morally fundamental for us or that it could simply be an attribute of God, alongside omniscience, omnipotence and perfect goodness, that God has authority. Indeed, one would expect that such authority is a part of perfection, and so there is a perfect being theology argument for God's having it. And given that western monotheism has such a strong focus on obedience to God, accepting some such view seems preferable to simply denying the duty.

7. FROM DAMNATION TO HELL

The Hell doctrine is stronger than the damnation doctrine in three ways: the cut-off for salvific chances happens at death, there is pain in hell, and there is eternal existence. I claim that we can add all three of these features to our two Damnation Toy Models without endangering their compatibility with God's character.

The easiest issue is the placement of the cut-off. We are simply giving a defense—an argument for logical compatibility—rather than trying to give a realistic model that matches how the world is. Suppose God foresees that a person about to die no longer has time to make any significantly free choice in this life. God then miraculously speeds up their functioning for the remaining fraction (let us suppose) of a second of their life, performing whatever other miracles are needed¹², in such a way that over that fraction of a second they live subjectively long enough to receive 3000 offers of Grace (and if supertasks are impossible God could even fit an infinite number of offers into that finite duration), while giving no more offers after death. If one of our Damnation Toy Models was compatible with God's goodness, generosity and respect for freedom, this story when appropriately filled out will also be compatible with it. But it is a story where death is the end of one's chances for salvation.

I already briefly argued that specifying that existence continues forever after the offers have run out can be defended by emphasizing the value of living by a character that has been chosen, by invoking the Augustinian

¹²Maybe the person is too brain damaged to make decisions. If so, this is fixed.

thesis that existence is always good, and inviting the reader to consider the possibility of some meaningful activities.

Existing forever, however, becomes intuitively more difficult to defend if we add that there is pain. But, again, we are giving a defense. For our compatibility claim, we need not argue that all the sufferings portrayed by Dante—or even the scriptures and traditions of the Western monotheistic traditions—are compatible with God’s character. We just need *some* endless suffering to be compatible with it. The damned, plausibly, deserve to suffer *some* pain. We might agree with Boethius [4] that it is intrinsically good for one to get what one deserves, even if receiving it is painful. But even if we do not agree with him, it is plausible that it is *just*. It is a part of the traditional theism of the three major western monotheistic religions that God’s goodness includes a justice according to which God metes out punishment, and it is difficult to deny that such retributive justice is compatible with God’s character. Furthermore, we need not suppose that the pain is *imposed* externally on the damned (i.e., those who have run out of offers of Grace). The persons could be naturally so constituted that they feel moderately painful guilt when they have culpably done wrong. Such a constitution in a creature appears compatible with a perfectly good creator.

If we add such a guilt-inducing natural constitution to our Damnation Toy Models, in addition to making the tweak to ensure the offers of Grace are prior to death, the result will be hell as long as we specify that any pleasures that may be present—say, pleasures in any potentially added meaningful activities—are insufficient given the pains to balance out the pains.

Thus, hell—and a non-empty one, given that the Models have about a thousand people expected to be damned—is compatible with God’s goodness and respect for our character.

Furthermore, we might suppose that those whose character is already Wicked are more likely to be motivated by avoidance of suffering than the attainment of beatitude. Thus, we could add to the Models that when God makes his offers, he commands the acceptance of Grace while promising pain for those who reject the offer. Since this could be expected to increase the probability of the acceptance of the offer, and thereby increase the probability of eternal beatitude for the person, the addition seems compatible with God’s goodness. But God is perfectly honest, and so he needs to follow through on the promise.

We might have one objection, however. Even if the pain in hell is moderate, if it lasts infinitely long, it seems that the total amount of pain over eternity will be infinite, and it is contrary to justice to punish a finite number of sins—such as the 3000 rejections of Grace—by an infinite punishment. One famous response here is St. Anselm’s: our sin imposes infinite guilt because it is an offense against an infinite God. A very different response, on the other hand, is that the doctrine of hell as I have defined it for this paper does not specify that the total amount of pain adds up to infinity. It is compatible with the doctrine of hell in the technical sense here that the pain decreases exponentially over time, and the sum (or integral) of a decreasing exponential is finite; for example, $1 + 1/2 + 1/4 + 1/8 + 1/16 + \dots$. Moreover, we need not suppose that the amount of pain decreases. We could instead suppose that the amount of pain remains constant, but there are some minor

pleasures that do not quite balance out the pain, but they come closer and closer to doing so, in such a way that the daily hedonic balance of pleasure minus pain, while always negative, adds up to a finite negative number.

Another possible move is to suppose that although the damned spend an infinite amount of objective time in hell, their subjective time in hell is finite, because their mental processing slows down over time, so that the first objective year in hell feels like a year, but the second objective year feels only like half a year, the third like a quarter, and so on, resulting in a subjectively finitely amount of time—and hence presumably a finite amount of suffering. Whether this is compatible with the doctrine of hell depends on whether “existing forever” in the doctrine is understood according to subjective or objective time.

8. CONCLUSIONS

There is a toy model of an account where God gives what appears to be a reasonable balance of the freedom to affect one’s character to creatures combined with an infinite number of chances at salvation, and this model appears compatible with God’s goodness, generosity, and respect for freedom. It is a mathematical consequence of the model that there is a non-zero probability of an agent continuing to reject God.

It is then possible to modify the model—either using a supertask or a merely finite cut-off—to yield models where the number of chances at salvation is finite, but where one can nonetheless argue that if the infinite chances model was compatible with God’s character, so is this one. This shows that the doctrine of damnation—that some persons are in a position where they

no longer have any chance at salvation—is compatible with God’s goodness and generosity. Finally, the model is modified to show that a doctrine of hell—at least of a somewhat moderate sort—is compatible with it as well.

One effective place for a reader to disagree would be in the first step, by requiring that a perfect God instead have a plan where it is probabilistically inevitable that everyone is saved, even if it might be metaphysically possible for some not to be. Whether this is so appears to depend on how much weight one puts on the value of the freedom of creatures to stay out of union with God. It is, I think, plausible that God can choose to balance the values in such a way as to allow a non-zero probability of the creatures always rejecting him. Alternatively, Molinism and theological compatibilism can undercut the first step.

But at least we still see the logical implication that if we allow that God can set up an infinite chances system with a non-zero chance of someone never being saved, we should also allow that God’s existence is compatible with the damnation doctrine, and likely also with the doctrine of hell.

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